Vitaly Efimov, Talk at 22nd International Conference on Few-Body Problems in Physics (FB22) in Caen, 11.07.18 at the ceremony on awarding Faddeev medal **Road to a Three-Boson Surprise**

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Abstract This is the text of the talk given at the XXII International Conference on Few-Body Problems in Physics, Caen, France, July 9-13, 2018. The author is a co-recipient of the 2018 Inaugural Faddeev Medal. First of all, I would like to thank the Few-Body Physics community for honoring my and Rudi Grimm's work with the first-ever Faddeev Medal. In my talk Iwill not spend much time on what we have done: in this audience I guess almost everybody knows what we have done.1 Instead I will focus on what you probably don't know. I will talk about a serpentine road that led to the 1970 finding of the surprising properties of quantum three-body systems [4,5]. On my way along this road I was fortunate to meet Ludvig Faddeev, and I will talk about this too.

To be followed easier, I break this road into ten segments.

1. The starting point is Leningrad, now St. Petersburg, 1955. By that time, upon graduating from a high school, I was sure I liked math and physics more than history or literature.

2. Following a suggestion by my father, mechanical engineer, I began college studies at Leningrad Electrical Engineering Institute. I soon realized that I greatly enjoyed higher math and physics, and I developed a habit of thorough independent study of these subjects. This gradually became my main occupation. Initially I considered my self-studies as groundwork for my future engineering career, but that reason faded with time. I graduated in 1962, earning the master degree in radio engineering.

3. The title of my master-degree thesis was "Statistical Approach to Ground States of Atomic Nuclei." The title and content of the thesis had little to do with radio engineering. Fortunately the professors relented and allowed the submission and presentation. The sharp turn from radio engineering to theoretical nuclear physics was initiated by a purely accidental event. About two years before the graduation, while walking in a street, I ran into Miron Amusia. He graduated from the same high school few years earlier than I. We knew each other quite well.2 At the time of our chance encounter Miron was a young theorist at the Theoretical Physics Division of the famous Ioffe Phys Tech Institute in Leningrad.3 As we walked along the street, I shared with him my aspirations. After several more meetings Miron made it possible for me to regularly visit seminars at the Ioffe Institute. He shaped the direction of my further self-studies and introduced me to his boss Lev Sliv, head of the Nuclear Theory group. After I presented my calculations involving electromagnetic resonators—I had done them working part-time at a Leningrad Electrical Engineering Institute lab—and after my master-thesis presentation, Sliv decided to accept me to his group. This way,

1 For a brief review of basic concepts see Efimov [6]; Ferlaino and Grimm [7]. Recent developments are reviewed by Naidon
and Endo [10]; Greene et al. [8]; D'Incao [2].

2 We first met when Miron was assigned to mentor a group of younger school kids including myself of age 10.

3 He is still there today; he is also at the Hebrew University, Jerusalem.

immediately after graduating from Leningrad Electrical Engineering Institute, I started at the Ioffe Institute. A self-taught beginner, I was absolutely happy!

4. Miron suggested the direction of my initial research, and in 1966 I earned the PhD degree presenting the thesis titled "Pair Collisions in a Low-Density Degenerate Fermi Gas." The main results were published in Annals of Physics [1].

5. There were two reasons I then decided to focus on the quantum three-body problem. First reason naturally emerged from my studies of low-density Fermi gas. At low densities the properties of the gas are determined by pair collisions of gas particles. As the density increases, the effect of triple collisions grows. One therefore needs to properly take it into account, which requires understanding of three-body physics. The second reason was no less important. I knew that the

three-body problem in classical and quantum physics was considered one of the toughest. In particular, the three-nucleon problem was not clearly understood. Being a quite ambitious and self-confident young man, I decided I was up to the challenge.

6. I started with the collision of three quantum hard spheres. I made this choice because my previous work on Fermi gas involved particles with a strong repulsive interaction imitating the strong short-range repulsion between the nucleons. To my amazement, I was able to solve this problem.[4] The result was published in Soviet Journal of Nuclear Physics [3].

7. The success with three hard spheres served as a great encouragement to continue working in the field. At that time a typical paper in nuclear or atomic three-body physics involved heavy numerical calculations, with not much of physical insight. A rare exception was the paper by Llewellyn Thomas [11], and I decided to carefully study it. Thomas showed that if two quantum particles are loosely bound, three of them will definitely be bound tightly—the size of their bound state will be on the order of the range of interparticle forces. This is called the Thomas collapse. Thomas'work had a significant impact on the early development of nuclear physics because his finding demonstrated that the range of nuclear forces cannot be too short. Indeed, the deuteron can be considered a loosely bound two-nucleon system. According to Thomas, if the range of nuclear forces were too short, the three-nucleon system, triton, would collapse, which is not the case. I asked myself the question, How will three-body forces affect the Thomas' result? After all, particles can interact not only pairwise but also when three of them closely approach one another. With this in mind, I modified the trial wave function constructed by Thomas so that it now included the effect of strong three body repulsion. This modification affected the Thomas' result very little as long as the ranges of two- and three-body forces were assumed to be of the same order. But a strange situation emerged when I made the range of three-body repulsion significantly greater than the range of two-body forces—but still less than the size of the loosely bound state of two particles. I expected that such strong and extended three-body repulsion would destroy the binding of three particles. To my surprise, it didn't. Puzzled, I considered an extreme case. I set the binding energy of two particles to zero so that the size of their loosely bound state became infinite. Keeping the three-body repulsion strong, I then began to gradually increase its range. I found that no matter how long this range was, the three-body bound state survived! I tortured myself with this striking result for quite a while until I realized[5] that (i) the loose two-body binding generates an effective long-range attraction of three particles, and that (ii) the range of the attraction is approximately equal to the size of loosely bound two-body state.

That was the key finding. A simple physical picture now emerged. Suppose a particle approaches a loosely bound pair. The particle will start feeling the presence of the pair at distances approximately equal to the size of the pair. At these distances one particle of the pair can leave the pair and form a similar pair with the incident particle. This process of particle exchange can repeat itself many times giving rise to an effective three-body attraction with the range equal to the size of the pair.

8. After the physics became clear, it was much easier to deduce unusual properties of the new phenomenon. It turned out that due to the long range of the effective three-body attraction, it was able to support numerous excited three-body states. In the extreme case when the binding energy of two particles is zero—so that the range of the three-body attraction gets infinite—the number of such excited states is infinite. They are all similar to one another differing only in their scale. Another interesting result was that with increase of the strength of two-body forces the number of those excited states may decrease. The physical picture explained this

4 To be more precise, I demonstrated that the problem can be reduced to a one-dimensional equation to be solved numerically.

5 A simple quantum-mechanical example helps see that a strong repulsion of arbitrarily long range does not necessarily destroy binding. Consider a particle bound by an attractive $1/r$ or $1/r^2$ potential cut at shorter distances by a repulsive core. The particle remains bound by such potential regardless of how large the core radius is.

paradox in simple terms: with strengthening of two-body forces, the binding energy of the pair increases, the pair tightens, and its size decreases causing the range of the three-body attraction to decrease. Such attraction supports less three-body bound states.

9. In Winter 1970 I completed the work and discussed it at a seminar of our group. Miron readily accepted and supported my findings. Soon after that, while pacing a hall of the Ioffe Institute building, I saw Ludvig Faddeev. Those days Faddeev was already a prominent figure in quantum physics. His workplace was the Leningrad branch of the Steklov Mathematical Institute. Yet he sometimes came to the Ioffe Institute to discuss his research. I knew him well because we previously talked about my paper on three hard spheres. Upon seeing him in the hall, I approached him and drew a diagram of three-body bound states on a tiny piece of paper (today this diagram is often referred to as the Efimov plot). I briefly explained the meaning of the diagram. Faddeev was very surprised. In a few days he called me and said he confirmed my results using his own method. Since then he firmly and actively supported my findings. As far as I know, he recommended publication of my paper in Soviet Journal of Nuclear Physics [4]. Miron encouraged me to write a brief report for Physics Letters. It was published [5] and became widely known.

10. The immediate reaction to those two papers was quite strong, yet mixed. Some people enthusiastically supported the results. Some others were openly skeptical. I heard a lot of different opinions. I also heard that a physicist who refereed my Physics Letters manuscript was initially certain that the results were wrong. Yet trying to disprove them, he came up with his own proof of their validity. Some skeptical opinions were published as recently as in 2013.

To conclude, looking back at this story, I could say it was a good mixture of passion, luck, and perseverance. “Chance favors the prepared mind,” famously said Louis Pasteur, and this story seems to fit his words nicely. I was extremely lucky to meet Miron Amusia, Ludvig Faddeev, and Lev Sliv at the right place and the right time.

I would add that I was also fortunate to witness the second life of my findings after the remarkable experiment by Rudi Grimm and his group [9]. But this is a different story.

References

1. M.Ya. Amusia, V. Efimov, Ann. Phys. (N. Y.) **47**, 377 (1968)
2. J.P. D’Incao, J. Phys. B **51**, 043001 (2018)
3. V. Efimov, Yad. Fiz. **10**, 107 (1969)
4. V. Efimov, Yad. Fiz. **12**, 1080 (1970)
5. V. Efimov, Phys. Lett. **33B**, 563 (1970)
6. V. Efimov, Few-Body Syst. **51**, 79 (2011)
7. F. Ferlaino, R. Grimm, Physics **3**, 9 (2010)
8. C.H. Greene, P. Giannakeas, J. Pérez-Ríos, Rev. Mod. Phys. **89**, 035006 (2017)
9. T. Kraemer et al., Nature (London) **440**, 315 (2006)
10. P. Naidon, S. Endo, Rep. Prog. Phys. **80**, 056001 (2017)
11. L.H. Thomas, Phys. Rev. **47**, 903 (1935)